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GEOMETRICAL METHOD AND DEVICE USED TO DRAW THE RESONANCE GRAPHICS FOR A RLC SERIES CIRCUIT IN ALTERNATIVE CURRENT

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Abstract. The paper form and develop the student's skills and abilities to measure indices of the alternating current (AC) in permanent regime. It develops experimental data-processing capacity by calculating reactance, impedance, AC power and plotting the resonance curve. It develops the ability to use units of measurement of electrical quantities as: impedance, reactance, active and reactive power. It develops the capacity of observation by highlighting side effects occurring in achieving RLC series resonance circuit voltage in (AC). Students determine the resonance phenomenon of RLC series AC circuit is characterized by:

- equal absolute values of tension on reactive circuit elements;
- maximum intensity of current through the circuit;
- equality of inductive reactance and capacitive reactance;
- circuit impedance is equal to its ohmic resistance;
- active power equals apparent power;
- circuit terminal voltage is equal with ohmic voltage.

Questions are meant to end pre-enhance knowledge formed by performing the work.

***Keywords*:** resistance, impedance, capacitor, reactance, Ohm's law, phasor diagrams.

Introduction

Feature size is either (AC) voltage or intensity, expressed by relations [HORTOPAN, 1980, IGNEA A., 1995]:

$$u = U_m \sin(\omega t + \varphi) \text{ and } i = I_m \sin(\omega t + \varphi'). \qquad (1)$$

Physical meanings of variables in Eq. (1) are: u and i - instantaneous values; U_m and I_m - maximum values; ω - pulsation; $\omega = 2\pi\nu$, ν - frequency, $\nu = \frac{1}{T}$, T - period; t – time; φ - initial phase.

The effective values of AC voltage and intensity are calculated by the relationship [FLOYD, 2010, UTRERAS-DIAZ, 2008]:

$$U_{eff} = U = \frac{U_m}{\sqrt{2}}; \quad I_{eff} = I = \frac{I_m}{\sqrt{2}}. \qquad (2)$$

AC circuit can contain the resistor, inductor and capacitor:

a) the resistor R current and voltage are in phase;

b) the coil with inductance L introduces an *inductive reactance* $X_L = \omega L$. Ideal coil inserts between current and voltage a phase angle $\varphi' = \frac{\pi}{2}$, current being out of phase behind from voltage. The voltage drop on the ideal coil is $U_L = IX_L$.

c) the capacitor with capacity C inserts *capacitive reactance* $X_C = \frac{1}{\omega C}$. On the capacitor the intensity is out of phase before of voltage with $\varphi'' = \frac{\pi}{2}$.

Series circuit impedance Z is calculated by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}. \qquad (3)$$

Ohm's law in effective values is:

$$I = \frac{U}{Z}. \qquad (4)$$

Phasor diagram of voltages is shown in *Figure 1* [DAMIAN, POPOV, 2002]. Relationship between voltages is:

$$U^2 = U_R^2 + (U_L - U_C)^2. \qquad (5)$$

Triangle impedance in RLC series AC circuit is shown in *Figure 2*.

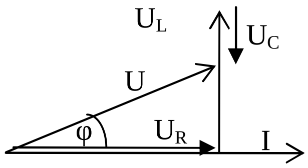

Figure 1. Phasor diagram of voltages.

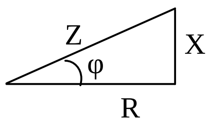

Figure 2. Impedance

Phase shift between current and voltage is:

$$\text{tg}\varphi = \frac{U_L - U_C}{U_R} \text{ or } \text{tg}\varphi = \frac{X_L - X_C}{R}. \qquad (6)$$

At resonance voltages, inductive and capacitive voltage drops are equal and

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opposite $U_L = -U_C$. Inductive and capacitive reactances $X_L = X_C$ are equal and minimum impedance is $Z = R$. The current through the circuit is maximum $I = \frac{U}{R}$.

AC powers are active, $P = RI^2 = UI\cos\varphi$, reactive, $Q = XI^2 = UI\sin\varphi$, apparent $S = ZI^2 = UI$ [FLOYD, 2010, UTRERAS-DIAZ, 2008].

Material and methods

The front panel of the installation is shown in Figure 3.

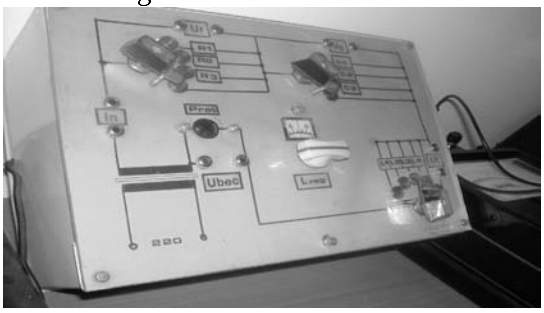

Figure 3. The front panel of the facility for the study RLC series circuit.

Electrical scheme of the facility for the study of RLC series AC circuit is shown in Fig.4.

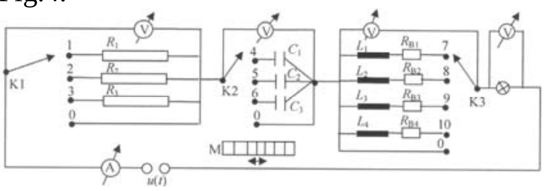

Figure 4. Electrical scheme of the circuit.

In Figure 4 we identify installation elements:

- resistors with resistances: R_1, R_2, R_3, $(R_{1,2,3})$;
- capabilities capacitors: C_1, C_2, C_3, $(C_{1,2,3})$;
- coils with inductances: L_1, L_2, L_3, L_4, $(L_{1,2,3,4})$ with ohmic resistances R_{B1}, R_{B2}, R_{B3}, R_{B4};
- circuit voltage is $U = 24$ V;
- K_1, K_2, K_3- switches introducing in the circuit the selected item: *R*, *L* or *C*;
- plots 1, 2, and 3 to connect resistors: $R_1 = 50\,\Omega$, $R_1 = 100\,\Omega$, $R_1 = 300\,\Omega$;
- plots 4, 5, and 6 to connect resistors: $C_1 = 1\,\mu F$, $C_2 = 5\,\mu F$, $C_3 = 10\,\mu F$;
- plots 7, 8, 9 and 10, to connect coils $L_1 \in (0.18 \div 0.9)$ H, $L_2 \in (0.73 \div 3.6)$ H, $L_3 \in (1.64 \div 8.2)$ H, $L_4 \in (2.18 \div 11.0)$ H.
- ruler *M* with slider to change inductance;
- bulb to avoid short-circuit;
- voltmeters to measure voltage circuit elements;
- terminals marked with zero (0) allow short-circuit of circuit elements.

The equipment allows the study of RL series circuit, RLC series RC or series.

Current intensity is measured with MAVO 35.

Voltage is measured with an electronic voltmeter E0401.

In the following tables are given values of circuit components.

Table 1.

Electrical resistance of the coils

Coil	B_1	B_2	B_3	B_4
$R_{B,i}(\Omega)$	67	142	228	272

Table 2.

Electrical resistance of the resistors

Resistor	R_1	R_2	R_3
$R_j(\Omega)$	50	100	300

Table 3.

Electrical capacities of the capacitors

Capacitor	C_1	C_2	C_3
$C_k(\mu F)$	1	5	10

Table 4.

Inductances of coils

Coil / Slider	B_1	B_2	B_3	B_4
A	0.18	0.73	1.64	2.18
B			1.65	2.20
C			1.66	2.22
A	0.19	0.74	1.67	2.23
B				2.24
C			1.68	2.25
A	0.19	0.75	1.69	2.26
B			1.70	2.28
C			1.71	2.30
A	0.18	0.77	1.73	2.32
B		0.78	1.74	2.34
C		0.79	1.79	2.39
A	0.20	0.81	1.80	2.44
B		0.82	1.84	2.49
C	0.21	0.84	1.89	2.55
A	0.22	0.86	1.95	2.60
B		0.88	1.98	2.70
C	0.23	0.92	2.05	2.80
A	0.24	0.95	2.16	2.90
B	0.25	0.99	2.24	3.00
C	0.26	1.02	2.30	3.12
A	0.27	1.06	2.45	3.23
B	0.28	1.13	2.55	3.50
C	0.31	1.23	2.81	3.79
A	0.36	1.44	3.20	4.30
B	0.43	1.79	3.80	5.83

C	0.58	2.40	5.56	7.70
A	0.83	3.49	7.75	10.50
B	0.88	3.55	8.00	10.80
C	0.89	3.57	8.15	10.90
A	0.90	3.57	8.20	11.00
B	0.90	3.60	8.20	11.00
C	0.90	3.60	8.20	11.00

Below are the operations to be carried out in pursuit of work:

1. Connect the ampermeter in series with circuit elements.
2. Connect the circuit elements *R*, *L*, *C*, selected with switches K_1, K_2, K_3.
3. Connect voltmeters at terminals of circuit elements.
4. Connect the installation at AC network.
5. *R* and *C* are maintained at selected values and with switch K_3 and ruler *M* modifies inductances. For each configuration of the circuit is measured voltages and current intensity. Repeat the previous operation for more three combinations of *R* and *C* elements and change on a wide range of inductance.

Note: To obtain an extended resonance curve on wide range of reactance values we initialize measurements such as: selecting *R*, *L* and *C* in switches K_1, K_2, K_3, then choose the range *a*, *b* or *c* and with ruler *M*, change inductance until current through the circuit is maximum. We denote inductance with L^*. For 10 proper inductance values higher than L^* and 10 values lower than L^* we measure the current and voltage through the circuit.

Results

Experimental data are processed following the model developed for R_1 and C_1 values kept constant and a variable inductance as current intensity increased to a maximum and then decrease. With measured values of current and voltages we calculate the terms presented below.

1. We calculate resistance $R_1 = \frac{U_R}{I}$ and bulb resistance $R_b = \frac{U_b}{I}$, and then we compare with the value of R_1 given in Table 2.
2. We calculate capacitive reactance $X_{C,1} = \frac{U_C}{I}$ and capacity $C_1 = \frac{1}{\omega X_{C,1}}$, $\omega = 2\pi\nu$, ν = 50 Hz, then compare C1 with the value given in Table 3.
3. Coil impedance wich is a RL series circuit is calculated using the formula $Z_B = \frac{U_B}{I}$. In order to determine the coil characteristics R_B and X_L we use the geometric phasors method. Short-circuiting the capacitor, note the terms with superscript prime and measure: $U_t = U'$, $U_b^{'}$, $U_R^{'}$, $U_B^{'}$, I'. Phasor diagram of voltages is shown in Figure 5. In Figure 5 we do not know $U_L^{'}, X_L, U_2^{'}, R_B$. We set the scale to represent phasors, i.e. if $U' = 24$ V and choose the scale of 0.5 cm / V phasors *U'* is the vector of a length of 12 cm.

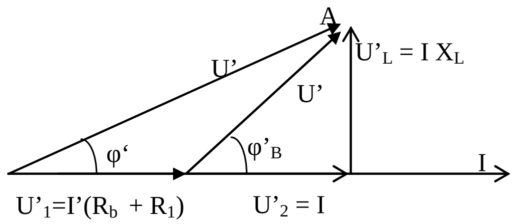

Figure 5. Phasor diagram: coil and resistor.

1. On the current axis (see *Figure 6*), we draw an arc C_1 of radius $r' = U'$ with center in point *O*. Phasor voltage U' intersects arc in a point to be found. We calculate and represent ohmic voltage phasor on resistor and bulb that sit along the reference axis until point B (*Figure 6*). Phasors coil drop voltage on $U_B^{'}$ intersect arc C_1 in the same point (A) as the terminal voltage phasor. [LUMINOSU, 2009, 2007]. We draw an arc centered on point B (Figure 5) with radius $r'' = U_B^{'}$ (arc C_2). Arcs C_1 and C_2 intersect in point A. The leg of perpendicular on the axis of the current determines phasor $U_2^{'} = I' R_B = |BD|$. The length of perpendicular from A to the axis of the current determines phasor $U_L^{'} = I' X_B = |DA|$. Given the scale of the representation we calculate R_B, X_L and *L*. We compare values that we find with the data found in Tables 1, 2 ,3 ,4.

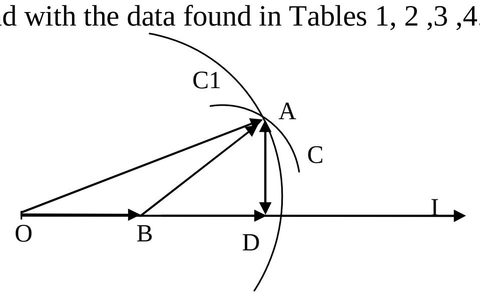

Figure 6. Diagram used at the geometric method.

4. The impedance of circuit is found with relation:

$Z = \frac{U}{I}$ and $Z^2 = (R_1 + R_B + R_b)^2 + (X_L - X_C)^2$.

5. Experimental data are summarized in Table 5.

We draw dependence $I = f(X_L)$ (*figure 7*), identifying the point of resonance (position 11 in Table 5) and make comments on the phenomenon of resonance characteristics.

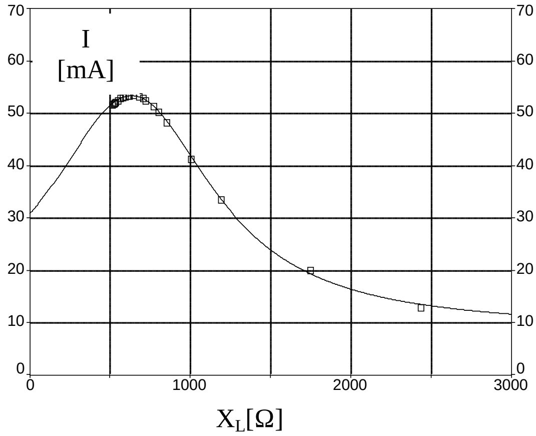

Figure 7. Current dependence of reactance.

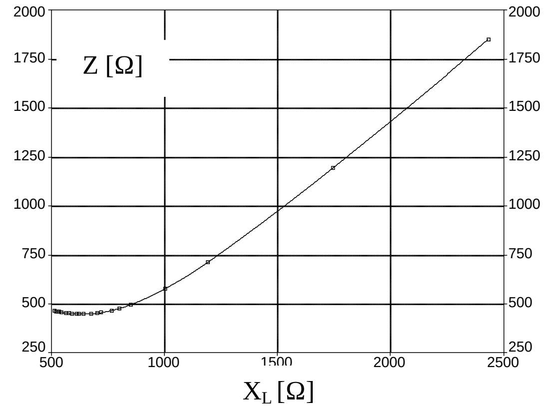

Figure 8. Impedance dependence of reactance.

The terms characteristic for resonance is synthesized in Table 6.

At the voltage resonance, the voltages on the capacitor and the coil are not equally, because the real coil is a series RL circuit.

The resonance is obtained when the X_L tends to become equal with X_C. At the resonance the system reactance X tends to zero but it is not equal with zero (see *Figure 8*) because the installation allows a step by step variation of resistances and capacitive and reactive reactances.

Table 5.

Series RLC circuit

No.	*I* (mA)	U_R (V)	U_C (V)	U_L (V)	*R* (Ω)	X_L (Ω)	X_C (Ω)	*C* (μF)	*L* (H)	*Z* (Ω)
1	51.7	23.2	32.9	26.6	448	514.9	636.9	5	1.64	464.3
2	51.9	23.2	33.0	27.0	448	521.2	636.9	5	1.66	462.7
3	52.0	23.3	33.2	27.5	448	527.5	636.9	5	1.68	461.0
4	52.2	23.4	33.3	27.9	448	533.8	636.9	5	1.70	459.7
5	52.5	23.5	33.4	28.7	448	546.4	636.9	5	1.74	457.0
6	52.98	23.7	33.7	29.9	448	565.2	636.9	5	1.80	453.7
7	53.1	23.8	33.8	30.7	448	577.8	636.9	5	1.84	451.9
8	53.3	23.9	33.9	31.6	448	593.5	636.9	5	1.89	450.0
9	53.5	23.9	34.1	32.8	448	612.3	636.9	5	1.95	448.7
10	53.5	24.0	34.1	33.3	448	621.7	636.9	5	1.98	448.3
11	**53.6**	**24.0**	**34.1**	**34.5**	**448**	**643.6**	**636.9**	**5**	**2.05**	**448**
12	53.3	23.9	36.2	34	448	678.2	636.9	5	2.16	449.9
13	53.0	23.7	37.3	33.7	448	703.4	636.9	5	2.24	452.9
14	52.6	23.4	38	33.5	448	722.2	636.9	5	2.30	456.0
15	51.4	23.0	32.7	39.5	448	769.3	636.9	5	2.45	467.2
16	50.3	22.5	32.0	40.3	448	800.7	636.9	5	2.55	477.0
17	48.3	21.7	30.8	44.1	448	850.9	636.9	5	2.81	496.5
18	41.4	18.5	26.4	41.6	448	1004.8	636.9	5	3.20	579.7
19	33.6	15.1	21.3	40.1	448	1193.2	636.9	5	3.80	714..3
20	20.1	9.0	12.8	35.0	448	1745.8	636.9	5	5.56	1195.9
21	13.0	5.8	8.3	31.5	448	2433.5	636.9	5	7.75	1851.6

Table 6.

Terms characteristic for resonance

$I_{rez.}$ (mA)	$X_{rez.}$ (Ω)	$Z_{rez.}$ (Ω)	$P_{rez.}$ (W)	$Q_{rez.}$ (VAR)	$S_{rez.}$ (VA)
53.6	6.7	448.1	1.29	0.02	1.30

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